Research on Teaching Evaluation of Senior High School Mathematics Classroom Driven by Problems

——Taking the Law of Cosines as an Example

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Abstract: The significance of mathematics classroom evaluation lies in ensuring that students understand and master mathematical concepts, methods, and problem-solving abilities. However, the existing evaluation of mathematics classroom teaching tends to emphasize learning results while ignoring the importance of the evaluation process itself. In problem-driven mathematics classrooms, evaluation examines mathematics textbooks from a more critical perspective. Therefore, in a problem-driven classroom environment, evaluation is even more important because it reflects students' understanding and abilities, helps teachers adjust teaching strategies, provides personalized support for students, and at the same time stimulates students' learning motivation and promotes their active participation in mathematics learning.

The Law of Cosines is not only a basic tool for solving triangle problems but also has extensive importance and application value in mathematical theoretical research and practical engineering applications. Therefore, it is necessary to conduct an evaluation study on problem-driven mathematics classroom teaching with the Law of Cosines as an example.

The mathematics classroom teaching of the Law of Cosines is divided into four links: introducing the Law of Cosines in a situational context, exploring and deriving the Law of Cosines, deeply understanding the Law of Cosines, and applying the theorem to solve problems. The 13 key problems in these links are evaluated and analyzed, and finally, reflections on classroom teaching evaluation are made.

Keywords: problem; classroom teaching evaluation; Law of Cosines

DOI:10.12417/3029-2328.25.08.019

1.Introduction

The classroom is the core of school education, teaching activities are the main way to impart knowledge and cultivate the pillars of the country, and teaching effect is also an important standard to measure excellent teachers [1]. Therefore, how to evaluate the quality of a lesson has become a long-term concern in the education field. Carrying out classroom evaluation activities is an important starting point for in-depth research on teaching practice, and it is of great significance for improving teachers' teaching strategies and teaching quality [2].

However, there are many problems in the existing evaluation of mathematics classroom teaching [3]. The function of evaluation tends to be selection and discrimination, and the purpose of evaluation mainly focuses on results. In addition, the evaluation subject is single, most of the evaluation methods are classroom communication and inquiry or written tests, and the evaluation mode is mainly summative evaluation [4]. This leads to classroom teaching and evaluation staying at the level of knowledge imparting, lacking in-depth understanding of students' mathematics learning process and courses.

In mathematics classrooms, asking questions is one of the effective ways to evaluate students' learning. The Law of Cosines is an important content in senior high school mathematics and a key examination point in the college

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entrance examination. Its importance in trigonometry is reflected in many aspects: it not only determines the relationship between the side lengths of a triangle, especially applicable to triangles of various shapes, but also can solve problems of non-right triangles. By calculating the relationship between any two sides and the included angle, for example, it can solve the area or side length. Therefore, taking the Law of Cosines in senior high school mathematics textbooks as an example, it is crucial to explore the effect of putting forward effective questions on evaluating students' learning.

2.Research Process

The following is a case of the Law of Cosines, taught by a pre-service graduate teacher from a normal university. The teaching textbook and chapter are the second section of the fourth section in Chapter 6 of the compulsory Volume 2 of the People's Education Press Version A.

2.1 Introducing the Law of Cosines in a situational context

Before teaching this lesson, the teacher first showed the students a video introducing a tunnel and told them that the length of the tunnel needs to be measured before excavation for equipment installation.

Question 1: How do engineers measure the length of the tunnel? If you were to excavate an expressway tunnel between places A and B at the foot of the mountain, how would you measure the distance between A and B?

Comment: Starting with practical applications in real life arouses students' interest in learning. Further, it brings students into a practical situation, asking them how to measure the length of the tunnel if they were to excavate it, which stimulates their curiosity and desire for knowledge, and at the same time enhances their sense of national pride.

The teacher first guides students to transform practical problems into specific mathematical problems.

Question 2: How to find the third side of a triangle given two sides and the included angle?

Comment: Instead of directly asking how to find the length of AB, the teacher adds the implicit side-angle relationship of the Law of Cosines. Given two sides and the included angle, students can associate it with the recently learned knowledge of the dot product of vectors. This way of asking not only paves the way for pointing out that the Law of Cosines derives the side-angle relationship but also guides students to connect old knowledge to solve problems and improves their knowledge transfer ability.

2.2 Exploring and deriving the Law of Cosines

The teacher further guides students to transform specific mathematical problems into more general situations.

Question 3: In triangle ABC, where the sides opposite angles A, B, and C are a, b, and c respectively, how to express c in terms of a, b, and angle C?

Comment: Connecting with Question 2, through the foreshadowing of Question 2, students can think of the dot product of vectors instead of being directly told the vector method. So far, through the foreshadowing of the first three questions, the solution to the problem is well guided to the vector method. Students go through a series of thinking processes, which is conducive to cultivating their habit of thinking and their ability to connect old knowledge.

Question 4: Combining with the research goal, what do you think should be done next?

Comment: It is not enough to just think of the dot product of vectors. It is also important to guide students to think of the properties of the dot product. Therefore, the teacher still uses questions and gives a hint of "research goal" in the question. Since it involves finding the modulus of a vector, coupled with the hint in the question, students can quickly think of the properties of the dot product on the basis of the dot product. Through step-by-step guided questions, students will more easily understand why to take the dot product of vector c with itself.

Question 5: What if we change the positions of the angle and the side?

Comment: The teacher leads students to derive only one of the formulas of the Law of Cosines. Since students have already explored and derived one formula, on the basis of their certain confidence, asking "What if we change the positions of the angle and the side?" can arouse their desire for knowledge, and then they can think independently to get the other two formulas of the Law of Cosines.

Question 6: Did you feel the power of vector operations in the process of deriving the Law of Cosines? Can you use your ability to derive the verbal expression of the Law of Cosines?

Comment: The two consecutive questions in Question 6: the first one implies the vector method to solve the problem; the second one, the teacher continues to adopt the previous strategy, on the basis of students' achievements, further stimulates their sense of competitiveness and desire for knowledge. Up to this point, through 6 key questions, students are driven to explore and obtain the formula and verbal expression of the Law of Cosines. Throughout the process, students are the main body of learning, their thinking is improved step by step, and they derive the core knowledge points through their own efforts, while the teacher plays a guiding, organizing, and prompting role.

2.3 Deeply understanding the Law of Cosines

The teacher first teaches the students the elements of the triangle involved in the Law of Cosines: therefore, using the Law of Cosines, we can find the third side of a triangle from the known two sides and the included angle.

Question 7: Can you associate it with the knowledge you learned before?

Comment: To introduce the relationship between the Law of Cosines and the triangle congruence model, it is necessary to first let students have a concept in mind that the Law of Cosines involves two sides and the included angle of a triangle. Then, asking students whether they can associate it with the previously learned knowledge, the question is concise and direct, and in fact, it also tells students that the Law of Cosines is closely related to old knowledge, further laying the groundwork for presupposing that students will think of the Law of Cosines later.

After deriving the relationship between the Law of Cosines and the congruence model, the teacher further points out that the Law of Cosines also derives the relationship between the three sides and angles of a triangle.

Question 8: How to determine an angle using the known three sides of a triangle [5]?

Comment: The teacher's prompt will arouse students' doubts, thus asking "How to determine an angle using the known three sides of a triangle?" drives students to observe the formula in combination with the problem, so as to derive the method of solving the problem by transposing terms.

Through solving Question 8, the teacher leads students to derive the corollary of the Law of Cosines.

Question 9: We know that when learning the Law of Cosines, it is the quantification of the SAS model. Then, which model is the corollary of the Law of Cosines the quantification of?

Comment: Before this, the teacher guided students to connect with old knowledge and derived that the Law of Cosines is the quantification of the SAS model. However, in this question, the teacher did not ask students to continue to connect with old knowledge but changed the way of asking, allowing students to have the awareness of connecting with old knowledge by themselves, thus improving their ability of analogical learning, knowledge transfer, and connecting with old knowledge.

By solving Question 9, students naturally conclude that the corollary of the Law of Cosines is the quantification of the SSS model.

Question 10: Observe the models for proving triangle congruence and the Law of Cosines. What are their differences?

Comment: Questions 7, 8, and 9 can be said to be the foreshadowing for Question 10. Through the previous three questions, students have a certain understanding of the relationship between the Law of Cosines and the congruence model, but they have not formed a systematic framework. Question 10 helps students form this framework. By guiding students to observe the differences, the teacher can then lead them to conclude that "trigonometric functions transform the qualitative conclusions about triangles in geometry into quantitatively calculable formulas". Thus, a knowledge system of the relationship between the Law of Cosines and the congruence model is constructed in students' minds.

2.4 Applying the theorem to solve problems

The teacher leads students to analyze the problem together. In the problem, given a, b, and sinC, finding angle B can be transformed into finding cosB or sinB.

Question 11: Do you have any further ideas?

Comment: Before asking this question, the teacher has led students to make a transformation, converting the target to be found into a target more closely related to the knowledge points, providing students with the idea and method of transformation. On this basis, letting students think, they can quickly associate it with the content learned in this lesson, thus step by step transforming the target to be found into cosC. At the same time, it makes students feel the process of backward reasoning and paves the way for deriving the backward reasoning method and problem-solving ideas later.

Question 12: How should cosC be found?

Comment: The idea of "needing to find cosC" can only be obtained through the backward reasoning method, so the problem becomes how to find cosC, which is connected with the teacher's previous analysis of the problem with students. The teacher has presupposed this question: some students fail to figure out how to find cosC because they miss the known condition sinC, while some students notice the known condition sinC in the process of analyzing the problem with the teacher, and then derive cosC.

Question 12: Observe these two problems. Which known elements of the triangle are they respectively using to find which unknown elements?

Comment: There are three types of triangle problems that can be solved by the Law of Cosines. If the teacher directly leads students to summarize, it is too abrupt, and students mechanically accept knowledge without thinking. The teacher notices this, so through questioning, guides students to try to summarize one type of problem by themselves, which paves the way for thinking about deriving the three types of problems later and improves students' ability to summarize problems.

Question 13: If a, b, and cosB are known in the formula, which elements can be found?

Comment: Through Question 12, it is concluded that the Law of Cosines can be used to solve the problem of finding the third side and the other two angles of a triangle given two sides and the included angle, so as to naturally derive that the Law of Cosines can be used to solve the problem of finding the three interior angles of a triangle given its three sides. Previously, students have only been exposed to the situation of knowing two sides and the included angle, and have not been exposed to the situation of knowing two sides and the opposite angle of one of them. Therefore, the teacher takes one of the situations as an example to ask, so that students can understand how to solve the last situation, ensuring the continuity of students' thinking and helping them further expand the previously constructed knowledge system of the Law of Cosines.

3.Reflection

"The essence of mathematics lies in freedom, and problems are the heart of mathematics [6]." Mathematics teaching should not be completely based on the framework of textbooks, but should make use of teachers' creativity to improve students' ability to discover and solve problems and enhance their core mathematical literacy. Problem-driven classroom teaching is crucial for teachers to create high-quality classes and improve teaching effects, especially in the context of the new curriculum reform, which has become a goal that every teacher should strive to achieve.

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